Coulsdon Sixth Form

Summer Homework Booklet

GCSE to A-Level

Key facts and formulae:

The Quadratic formula:

The solution of $ax^2 + bx + c = 0$

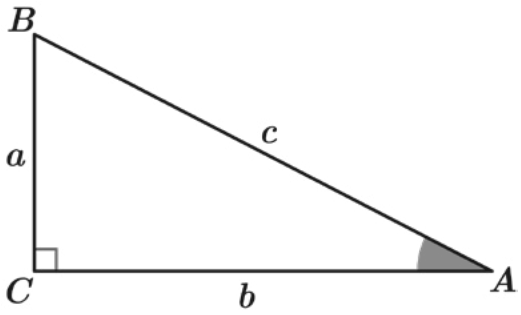
where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry:

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

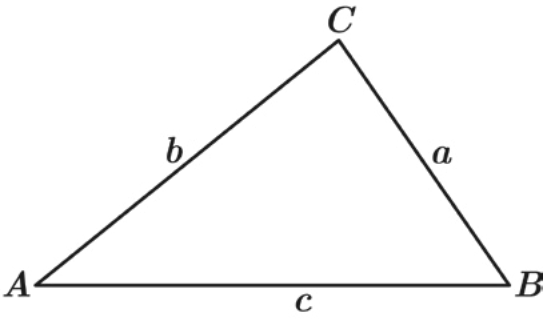
$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$



Q1 Expand and fully simplify $\sqrt{5}(\sqrt{5}+\sqrt{7})$

Answer: .....

Q2 Rationalise the denominator of $\frac{2\sqrt{5}}{\sqrt{6}}$

Give your answer in its simplest form.

Answer: .....

Q3 Expand and fully simplify $(6+\sqrt{5})(1+\sqrt{5})$

Answer: .....

Q4 Write $(5 + \sqrt{12})(11 + \sqrt{3})$ in the form $a + b\sqrt{3}$, where a and b are integers.

Answer: .....

Q5 Rationalise the denominator of $\dfrac{1 + \sqrt{2}}{\sqrt{2}}$

Give your answer as a fraction in its simplest form.

Answer: .....

Q1 Expand and fully simplify $(2\sqrt{6} - 5\sqrt{2})^2$

Answer: .....

Q2 Rationalise the denominator of $\dfrac{15 + \sqrt{3}}{10\sqrt{3}}$

Give your answer as a fraction in its simplest form.

Answer: .....

Q3 Rationalise the denominator of $\dfrac{2\sqrt{7}}{3+\sqrt{7}}$

Give your answer in its simplest form.

Answer: .....

Q4 Write $\sqrt{12} + \dfrac{33}{\sqrt{3}}$ in the form $r\sqrt{3}$, where r is an integer.

Answer: .....

Q1 Expand and fully simplify $(4 + \sqrt{7})^2 - (4 - \sqrt{7})^2$

Answer: .....

Q2 Work out the value of x in the equation below.

$x(\sqrt{11} - 2) = 21$

Give your answer in the form $a + b\sqrt{11}$, where a and b are integers.

Answer: .....

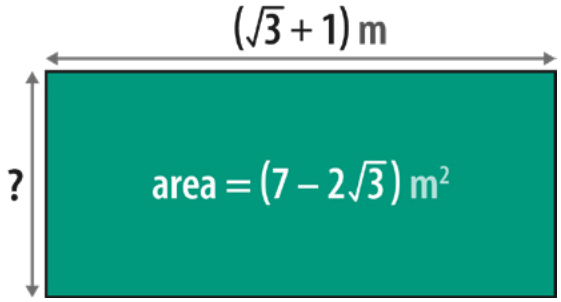
Q3 Given that h is a prime number, rationalise the denominator of $\dfrac{5h - \sqrt{h}}{\sqrt{h}}$

Give your answer in its simplest form.

Answer: .....

Q4 Calculate the unknown side length, in metres, of the rectangle below.

Give your answer in its simplest form, rationalising the denominator if necessary.



Answer: ..... m

Q1 Expand and fully simplify $(m + 9)(m + 2)$

Answer: .....

Q2 Expand and fully simplify $(2a + 3)(4a + 5)$

Answer: .....

Q3 Expand and fully simplify $(x - 3)(4x + 9)$

Answer: .....

Q4 Expand and fully simplify $(6n - 5)^2$

Answer: .....

Q1 Expand and fully simplify $2(4d + 5)(3d + 1)$

Answer: .....

Q2 Expand and fully simplify $(x + 1)(x^2 + 3x + 5)$

Answer: .....

Q3 Expand and fully simplify $(3n + 4)(5n + 2) + 5(n + 7)$

Answer: .....

Q4 Expand and fully simplify $(t - 2)(t + 5)(t - 4)$

Answer: .....

Q1 Expand and fully simplify $(2x + 5)(4x - 3)(5x - 4)$

Answer: .....

Q2 Work out the values of a, b and c in the identity below.

$(3x - 1)(x + 2)(ax + b) \equiv 15x^3 + 16x^2 - 25x + c$

Answer: a = ... b = ... c = ...

Q3 Write the following expression in the form $\frac{1}{ax^b} + \frac{1}{cy^d}$ where a, b, c, and d are integers.

$$\left(\frac{1}{5x} + \frac{1}{4y}\right)\left(\frac{1}{25x^2} - \frac{1}{20xy} + \frac{1}{16y^2}\right)$$

Answer: .....

Q4 Show that $(x^2 + 1)(y^2 + 4) \equiv (xy - 2)^2 + (2x + y)^2$

Q1 Fully factorise $y^2 + 9y + 20$

Answer: .....

Q2 Fully factorise $x^2 - x - 20$

Answer: .....

Q3 Fully factorise $w^2 - 15w + 54$

Answer: .....

Q1 Fully factorise x^2 - 16

Answer: .....

Q2 Fully factorise $2r^2 + 15r + 7$

Answer: .....

Q3 Fully factorise $5x^2 + 22x + 8$

Answer: .....

Q1 Fully factorise $49h^2 - m^2$

Answer: .....

Q2 Fully factorise $7b - b^2 - 10$

Answer: .....

Q3 Fully factorise $4k^2 - 25n^2 - (2k - 5n)^2$

Answer: .....

Q1 Fully simplify the expression $4y^5 \times 3y^2$

Answer: .....

Q2 Simplify $(h^{-5})^3$

Give your answer without any negative indices.

Answer: .....

Q3 Write $\frac{2t^6u}{8t^3}$ as a fraction in its simplest form.

Answer: .....

Q4 Fully simplify $\left(\frac{t^3}{u^5}\right)^2$

Answer: .....

Q5 Write $\frac{33xy + 9x}{18x}$ as a fraction in its simplest form.

Answer: .....

Q6 Fully simplify $\frac{6a + 42}{a^2 + 11a + 28}$

Answer: .....

Q1 Write $\frac{(3a)^2}{54ak}$ as a fraction in its simplest form.

Answer: .....

Q2 Fully simplify $(64g^8h^4)^{\frac{1}{2}}$

Answer: .....

Q3 Fully simplify $\frac{x+2}{2x^2-31x-70}$

Answer: .....

Q1 Work out the values of a, b and c in the equality below.

$$\frac{2x^{20}y^4 \times 12x^4y^{26}}{(2xy^2)^3} = ax^by^c$$

Answer: $a =$ ..... $b =$ ..... $c =$ .....

Q2 Work out what expression should replace the ? in the equivalent fractions below.

$$\frac{?}{12r^4(t+6)} = \frac{2n}{3r}$$

Answer: ? = ...

Q3 $\frac{ax^2 + bx + c}{dx^2 - 25}$ simplifies to give $\frac{x-4}{2x-5}$

Work out the values of a, b, c and d in the original fraction.

Answer: $a =$ ..... $b =$ ..... $c =$ ..... $d =$ .....

Q1 Fully simplify $\frac{14a}{b} \times \frac{b}{2}$

Answer: .....

Q2 Fully simplify $\frac{6a}{v} \div \frac{2a}{5}$

Give your answer as a fraction.

Answer: .....

Q3 Fully simplify the expression below to give a single fraction.

$$\frac{n+2}{5} + \frac{6n}{7}$$

Answer: .....

Q1

Fully simplify $\frac{2}{5a+4} \times \frac{45a+36}{a}$

Give your answer as a fraction.

Answer: .....

Q2

Fully simplify $\frac{6x}{(5x-7)(x+1)} - \frac{1}{5x-7}$

Give your answer fully factorised.

Answer: .....

Q3 Write the following as a single fraction in its simplest form:

$$\frac{2x^2 - 11x + 12}{x + 5} \div (4x^2 - 6x)$$

Give your answer fully factorised.

Answer: .....

Q4 Fully simplify $\frac{4ab^2}{k} \times \frac{3ak}{12k} \times \frac{7}{5ab}$

Give your answer as a fraction.

Answer: .....

Q1 Fully simplify $\frac{7}{36 - x^2} - \frac{3}{6 + x}$

Give your answer fully factorised.

Answer: .....

Q2 Write the following as a single fraction in its simplest form:

$$6 - (x + 4) \div \frac{x^2 + 11x + 28}{x - 7}$$

Give your answer fully factorised.

Answer: .....

Q1 Find the two solutions to the equation

$(x - 9)(x + 5) = 0$

Answer: .....

Q2 Solve this equation by factorising:

$y^2 + 3y - 10 = 0$

Answer: .....

Q3 Solve this equation by factorising:

$12 - 8w + w^2 = 0$

Answer: .....

Q4 Using the quadratic formula, solve

$4x^2 + 16x + 15 = 0$

Answer: .....

Q5 Solve this equation by factorising:

$2m^2 - 11m + 5 = 0$

Answer: .....

Q1 Using the quadratic formula, solve $y^2 - 6y + 7 = 0$

Give your answer in the form $a \pm \sqrt{b}$

Answer: .....

Q2 Solve the equation below using factorising.

$6y^2 - 11y - 10 = 0$

Answer: .....

Q3 Using the quadratic formula, solve $6x^2 - 35 = -11x$

Answer: .....

Q4 Solve $3r(3r - 4) = 2$

Give your answers to 2 d.p.

Answer: .....

Q1 Solve $x(x + 4) - 4(5x + 9) = 0$

Answer: .....

Q2 Jessica thinks of a positive number, n, which is less than 1
She adds this number to its reciprocal and gets 2.9

Work out the value of n.
Give your answer as a fraction in its simplest form.

Answer: .....

Q3

Solve $\frac{4}{y-1} - \frac{5}{y+2} = \frac{3}{y}$

Answer: .....

Q4

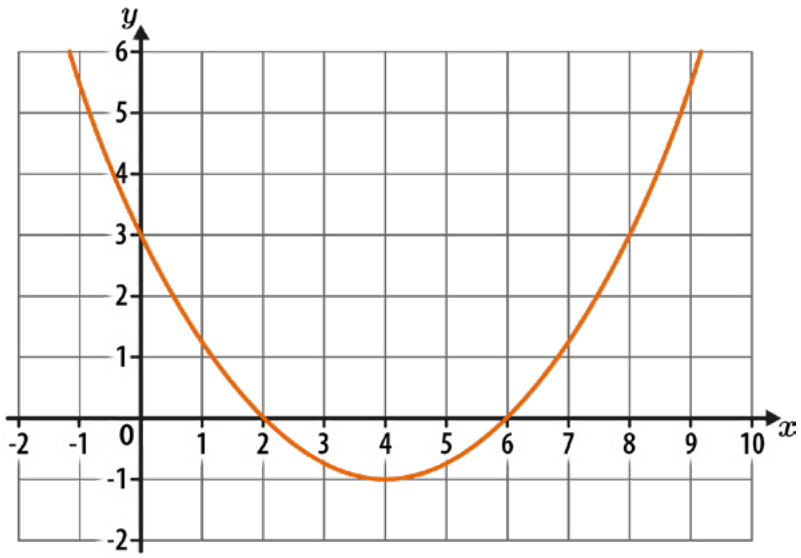
$x = \frac{-3 \pm \sqrt{29}}{2}$

There is only one equation of the form $x^2 + bx + c = 0$ that gives these values of x as solutions.

Work out the values of b and c.

Answer: $b =$ ..... $c =$ .....

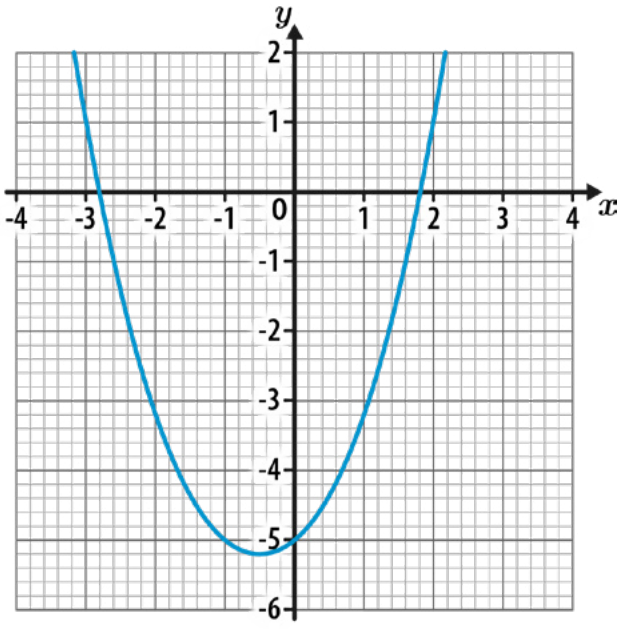
Q1 Write down the coordinates of the roots of the quadratic curve shown below.



Answer: (..... , ...) and (..... , ...)

Q2 Here is the graph of the function $y = x^2 + x - 5$

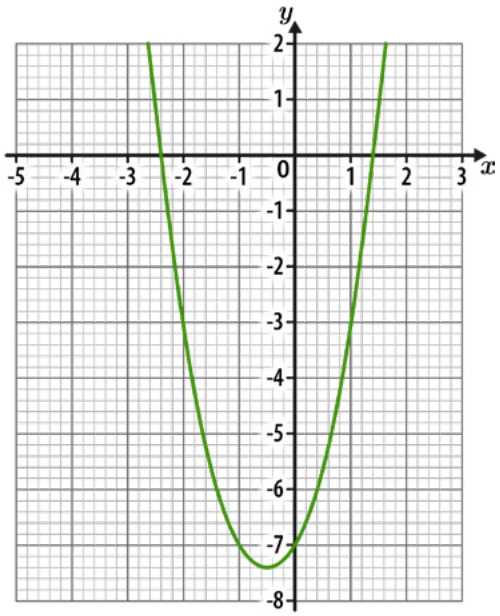
Estimate the solutions to $x^2 + x - 5 = 0$
Give your answers to 1 d.p.



Answer: .....

Q3 The diagram below shows the graph of the function $y = 2x^2 + 2x - 7$

Work out the solutions to $2x^2 + 2x - 7 = -3$



Answer: .....

Q4 a) Write $x^2 + 6x + 11$ in the form $(x + c)^2 + d$, where c and d are numbers.

Answer: a) .....

b) Hence, write down the coordinates of the turning point on the curve $y = x^2 + 6x + 11$

Answer: b) (..... , ...)

Q1 The diagram below shows a sketch of the curve $y = x^2 + 8x - 10$

P is the turning point of the curve.

Work out the coordinates of P.

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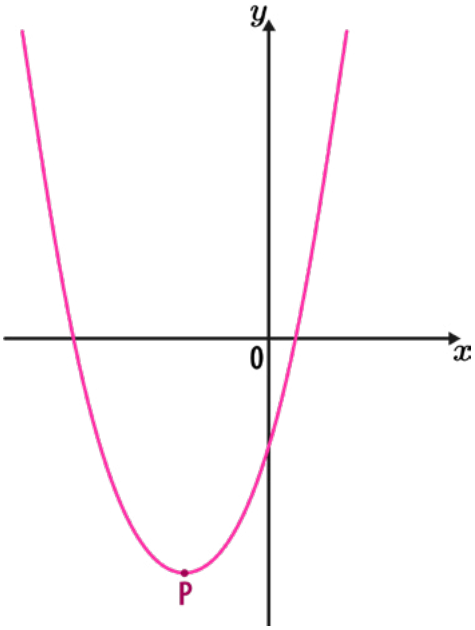


Answer: (..... , ...)

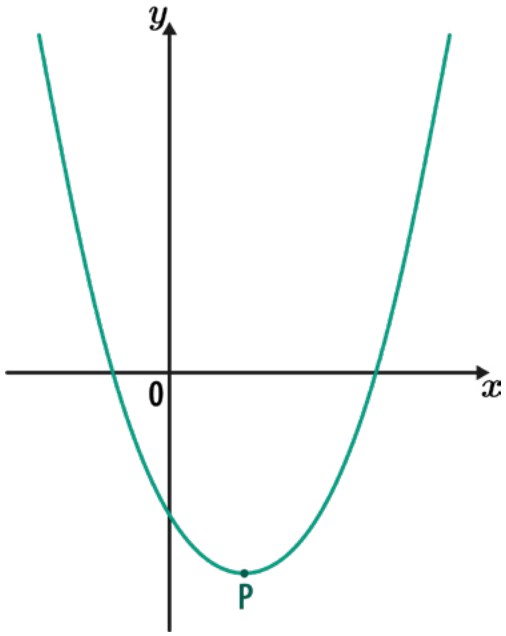
Q2 Work out the coordinates of the turning point of the curve $y = x^2 - 5x + 1$

Answer: (..... , ...)

Q3 The diagram below shows a sketch of the curve $y = 3x^2 - 6x - 10$

P is the turning point of the curve.

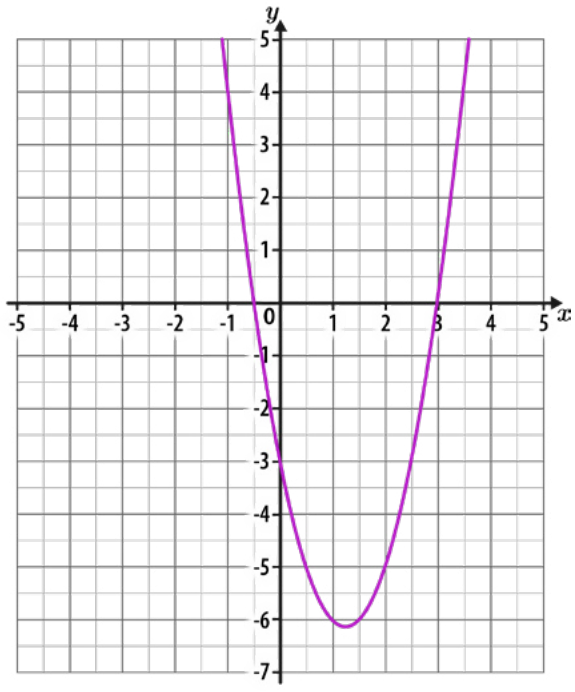
Work out the coordinates of P.



Answer: (..... , ...)

Q4 The diagram below shows the graph of $y = 2x^2 - 5x - 3$

Use the diagram to estimate the solutions to $2x^2 - 5x - 3 = -2x + 2$
Give any decimal answers to 1 d.p.

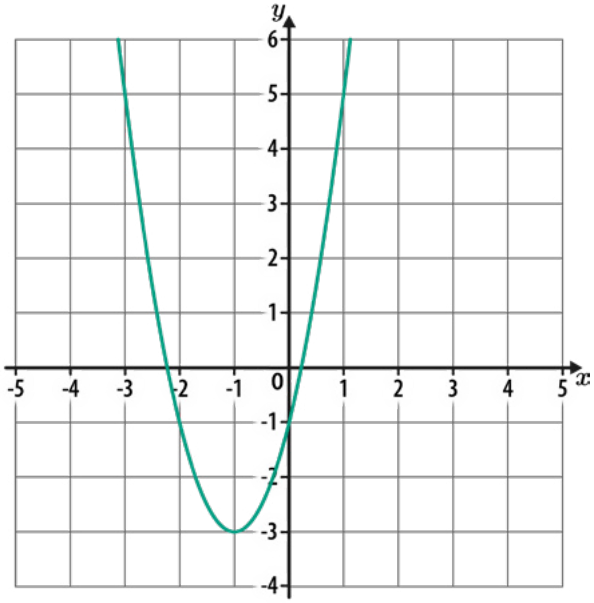


Answer: .....

Q1

The diagram below shows the graph of $y = 2x^2 + 4x - 1$
The equation $2x^2 + 4x - 1 = k$ has solutions at $x = -3$ and $x = 1$

What is the value of k?



Answer: $k =$ .....

Q2

A curve has the equation $y = x^2 + ax + b$, where a and b are numbers.
The turning point of the curve is (5, 4)

Work out the values of a and b.

Answer: $a =$ ..... $b =$ .....

Q3 A curve has the equation $y = -x^2 + 16x - 65$

a) Work out the turning point of the curve.

Answer: a) (....., ...)

b) By considering the position of the turning point and the shape of the curve, work out how many real roots $y = -x^2 + 16x - 65$ has.

Answer: b) .....

Q1 Solve the following simultaneous equations:

$6x + y = 22$

$2x + y = 10$

Answer: $x =$ ..... $y =$ .....

Q2 Solve the following simultaneous equations:

$7x - 4y = 20$

$2x + 4y = 16$

Answer: $x =$ ..... $y =$ .....

Q3 Solve the following simultaneous equations:

$15a - 4b = 25$

$5a + 2b = 25$

Answer: $a =$ ..... $b =$ .....

Q4 Solve the following simultaneous equations:

$2x + 3y = 8$

$3x + 4y = 11$

Answer: $x =$ ..... $y =$ .....

Q1 Solve the following simultaneous equations:

$7x + 5y = 8$

$3x - 2y = -9$

Answer: $x =$ ..... $y =$ .....

Q2 Solve the following simultaneous equations:

$6x + 7y = 5$

$9x + 13y = -10$

Answer: $x =$ ..... $y =$ .....

Q3 Solve the following simultaneous equations:

$7y + 2x = \frac{23}{2}$

$5y + 3x = 9$

Answer: $x =$ ..... $y =$ .....

Q4 Solve the following simultaneous equations:

$4.6t + 8.1u = 104$

$3.8t - 2.7u = -8$

Answer: $t =$ ..... $u =$ .....

Q1 Solve the following simultaneous equations:

$3x = 3 - 4y$

$12y + 11 = -5x$

Answer: $x =$ ..... $y =$ .....

Q2 Find the values of x, y and a by solving the following simultaneous equations:

$6x - 7y = -10$

$12x - 5y = 16$

$2x + ay = 10$

Answer: $x =$ ..... $y =$ ..... $a =$ .....

Q3 Solve the following simultaneous equations:

$$\frac{4}{7x - 4} = \frac{1}{6y}$$

$$\frac{5x}{3y + 2} = 4$$

Answer: $x =$ ..... $y =$ .....

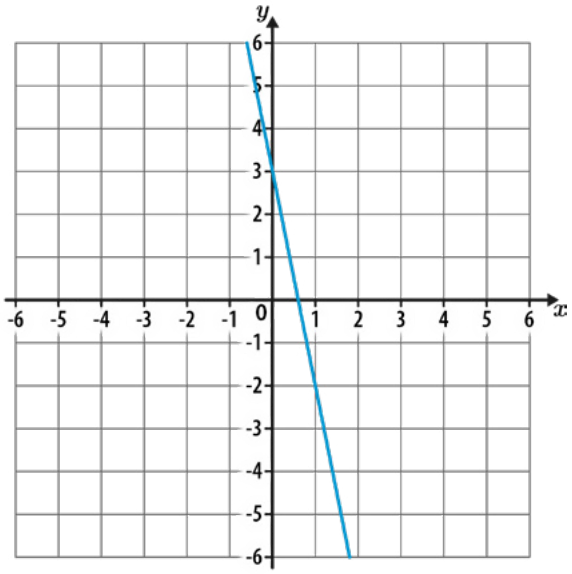
Q4 Solve the following simultaneous equations:

$$2^{x} = 4^{(7 - 2y)}$$

$$3^{(5x - 13y)} = 81$$

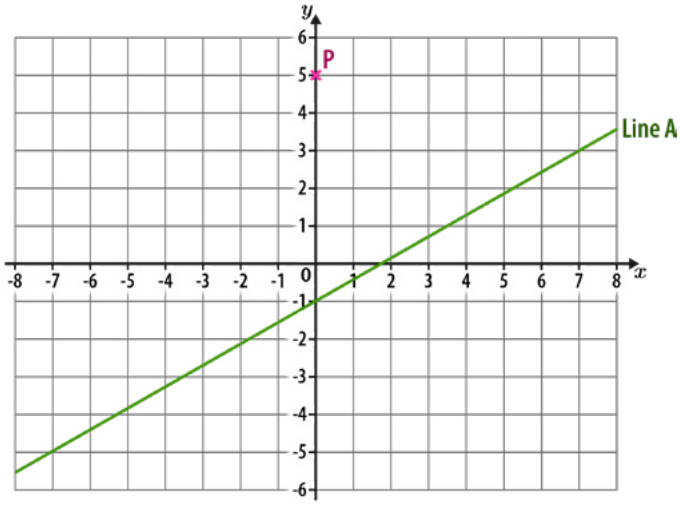
Answer: $x =$ ..... $y =$ .....

Q1 Work out the equation of the straight line shown below.



Answer: .....

Q2 Work out the equation of the straight line that is parallel to line A and passes through point P.



Answer: .....

Q3 Line A has the equation $2y - 10 = 16x$
Line B is perpendicular to Line A.

What is the gradient of Line B?

Answer: .....

Q4 A straight line has a gradient of 3 and passes through the point (2, 10)

Work out the equation of the line.

Answer: .....

Q5 Work out the equation of the straight line that passes through (2, 3) and (5, 18)

Answer: .....

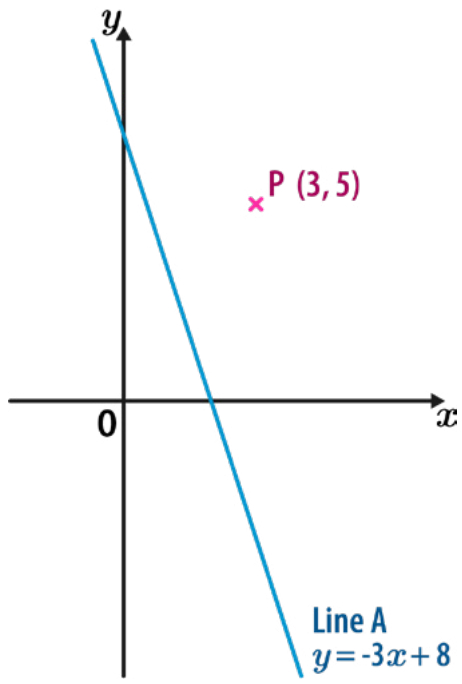
Q1 A straight line has a gradient of $-\frac{3}{4}$, and passes through the point (32, 12)

Work out the equation of the line.

Answer: .....

Q2 The diagram below shows point P and Line A.
Line B is **perpendicular** to line A and passes through point P.

What is the equation of line B?



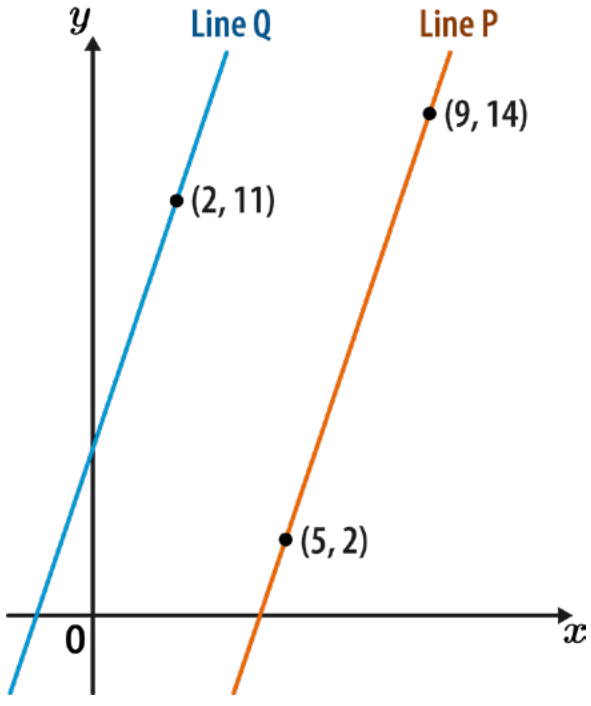
Answer: .....

Q3 Work out the equation of the straight line that passes through (1, -7) and (6, 8)

Answer: .....

Q4 The graph below shows line P and line Q.
Line Q is **parallel** to line P.

What is the equation of line Q?



Answer: .....

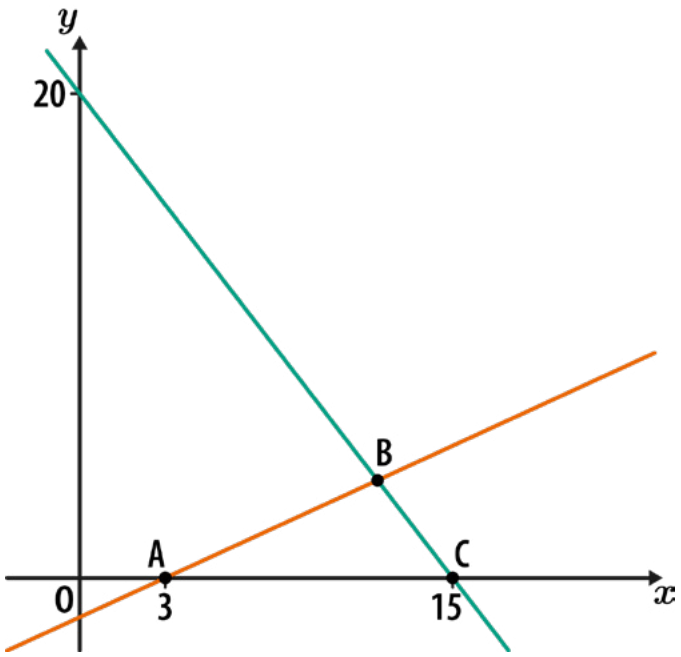
Q1 Write an expression, in terms of h, for the gradient of a line **perpendicular** to the line segment joining $(3h, 20)$ to $(6h, 8)$

Give your answer as a fully simplified fraction.

Answer: .....

Q2 The triangle ABC has an area of 24 square units.

What are the coordinates of point B?

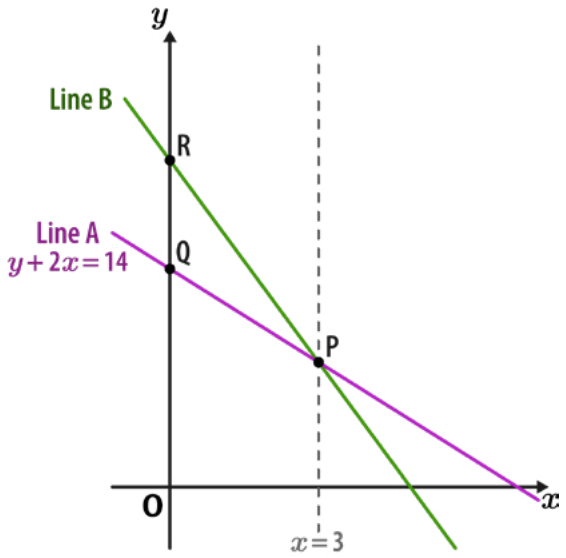


Answer: (..... , ...)

Q3

Line A has the equation $y + 2x = 14$
The gradient of line B is twice the gradient of line A.

Work out the ratio of the length of OQ to the length of OR.
Give your answer in its simplest form.

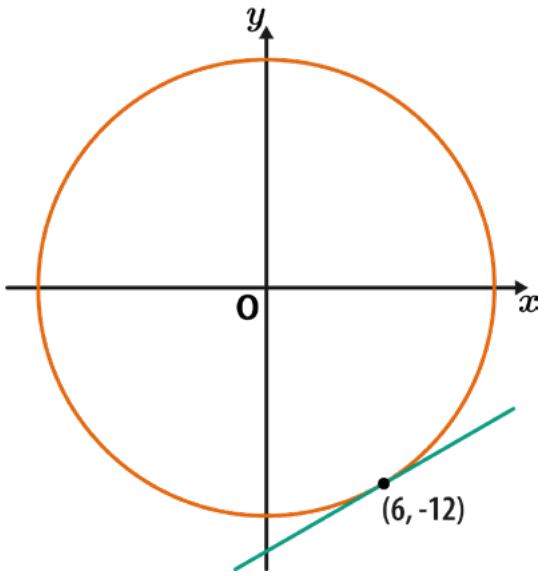


Answer: .....

Q4

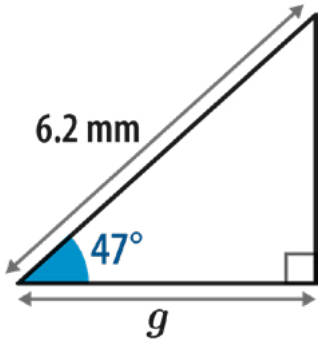
A circle, centre O, passes through the point (6, -12), as shown.

Work out the equation of the tangent to the circle at this point.
Give your answer in the form $y = mx + c$, where m and c are integers or fractions in their simplest form.



Answer: .....

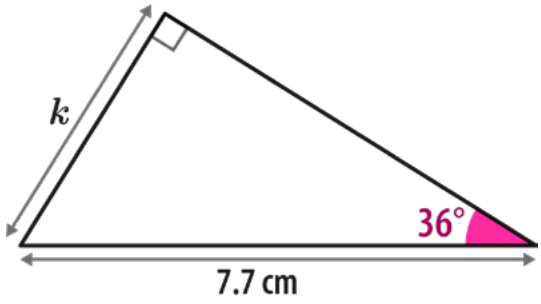
Q1 Work out the length g.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... mm

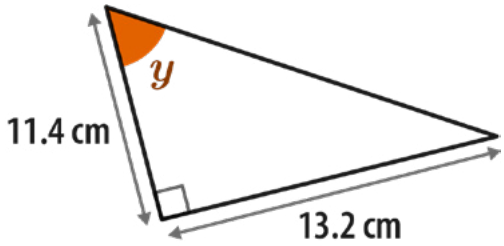
Q2 Work out the length k.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... cm

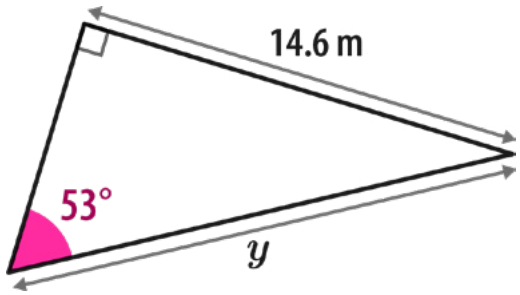
Q3 Calculate the size of angle y.
Give your answer to the nearest integer.



Not drawn accurately

Answer: ..... °

Q1 Calculate the length y.
Give your answer to 2 d.p.



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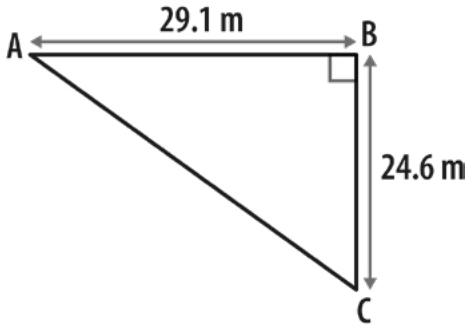
Answer: ..... m

Q2 Calculate the size of angle BAC.
Give your answer to 1 d.p.



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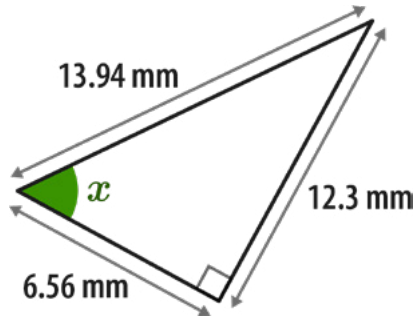
Answer: ..... °

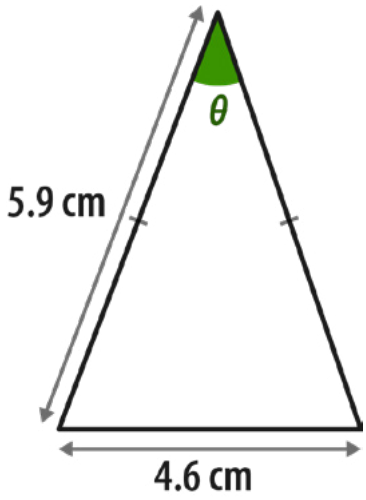
Q3 What is the size of angle x?
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... °

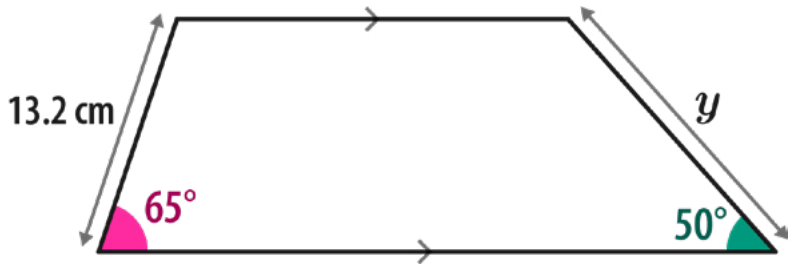
Q1 Calculate the size of angle θ.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... °

Q2 Work out the length y.
Give your answer to 2 d.p.



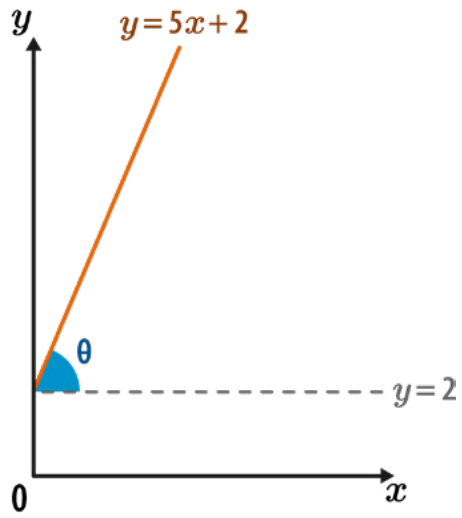
Not drawn accurately

Answer: ..... cm

Q3

The graph below shows the line with equation $y = 5x + 2$
The axes both have the same scale.

Calculate the size of angle θ.
Give your answer in degrees to the nearest integer.



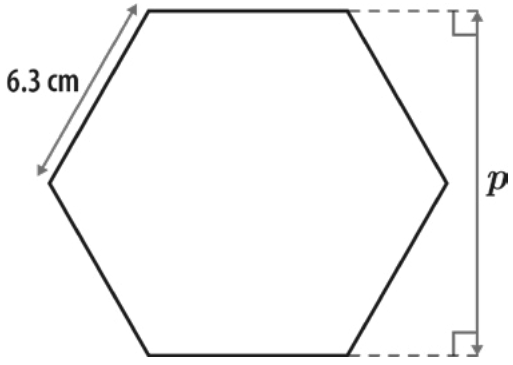
Not drawn accurately

Answer: ..... °

Q4

The shape below is a regular hexagon.

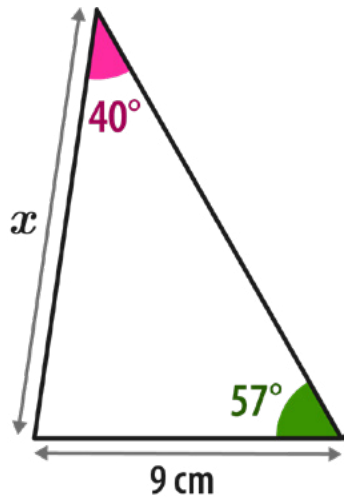
Use trigonometry to calculate the distance p.
Give your answer in centimetres to 2 d.p.



Not drawn accurately

Answer: ..... cm

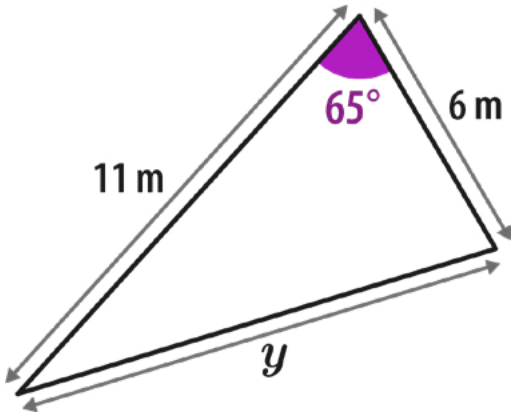
Q1 Using the sine rule, calculate the length x.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... cm

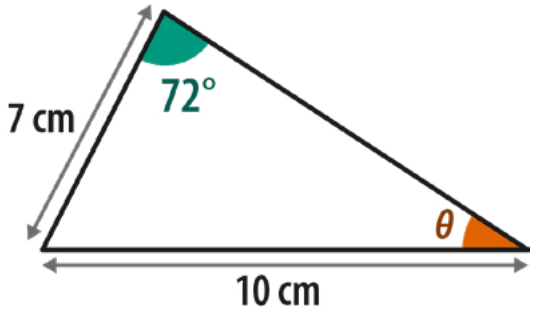
Q2 Using the cosine rule, work out the length y.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... m

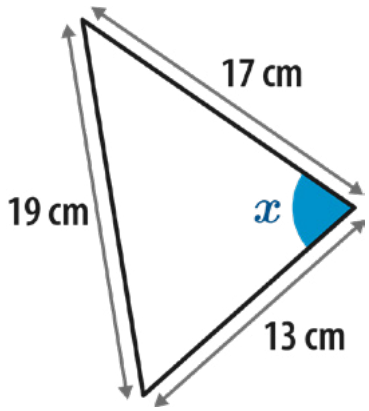
Q3 Use the sine rule to calculate angle θ.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... °

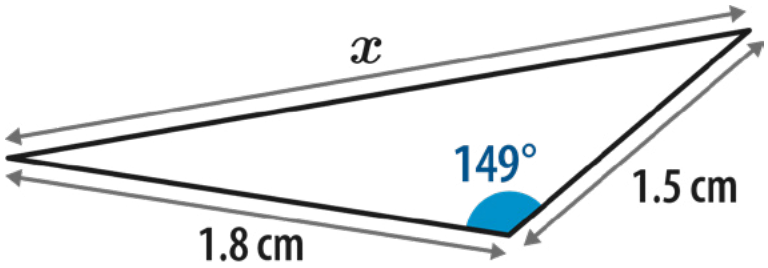
Q4 Use the cosine rule to calculate the size of angle x.
Give your answer to the nearest degree.



Not drawn accurately

Answer: ..... °

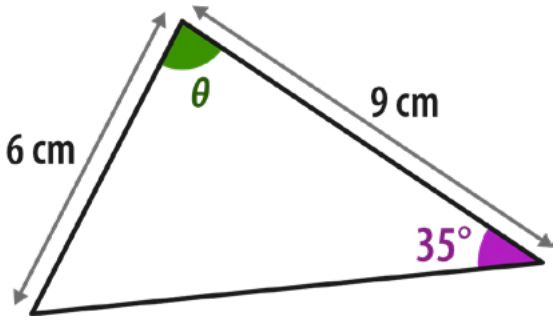
Q1 Work out length x.
Give your answer to 1 d.p.



Not drawn accurately

Answer: ..... cm

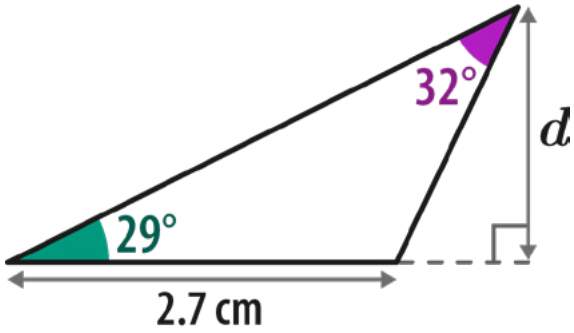
Q2 All the angles in the triangle below are acute.
Calculate the angle θ to 1 d.p.



Not drawn accurately

Answer: ..... °

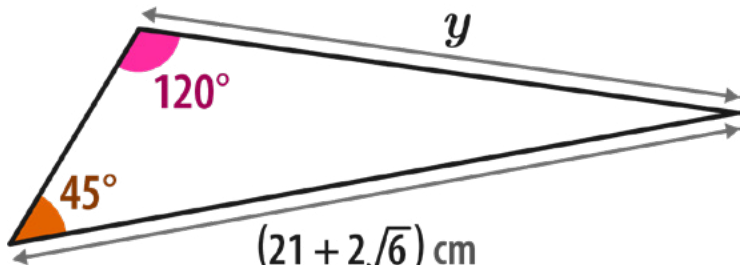
Q1 Calculate the length d.
Give your answer to 2 s.f.



Not drawn accurately

Answer: ..... cm

Q2 Work out the length y in the triangle below.
Give your answer in its simplest form, rationalising the denominator if necessary.

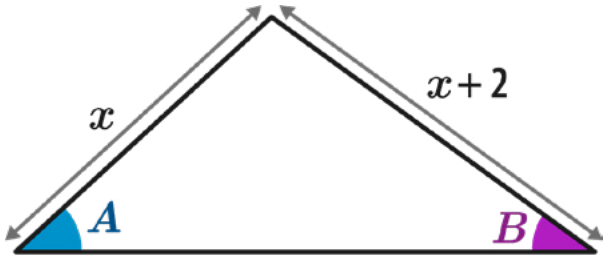


Not drawn accurately

Answer: ..... cm

Q3 Using the information below, work out the value of x.

$\sin A = \frac{4}{5}$ $\sin B = \frac{3}{4}$

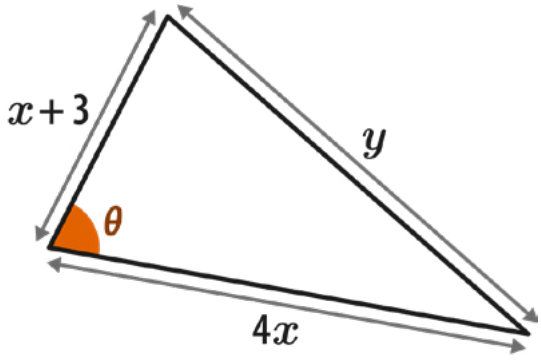


Not drawn accurately

Answer: .....

Q4 Given that $\cos\theta = \frac{1}{8}$ in the triangle below, show that $y^2 = ax^2 + bx + c$ where a, b and c are numbers.

What are the values of a, b and c?



Not drawn accurately

Answer: a = ... b = ... c = ...